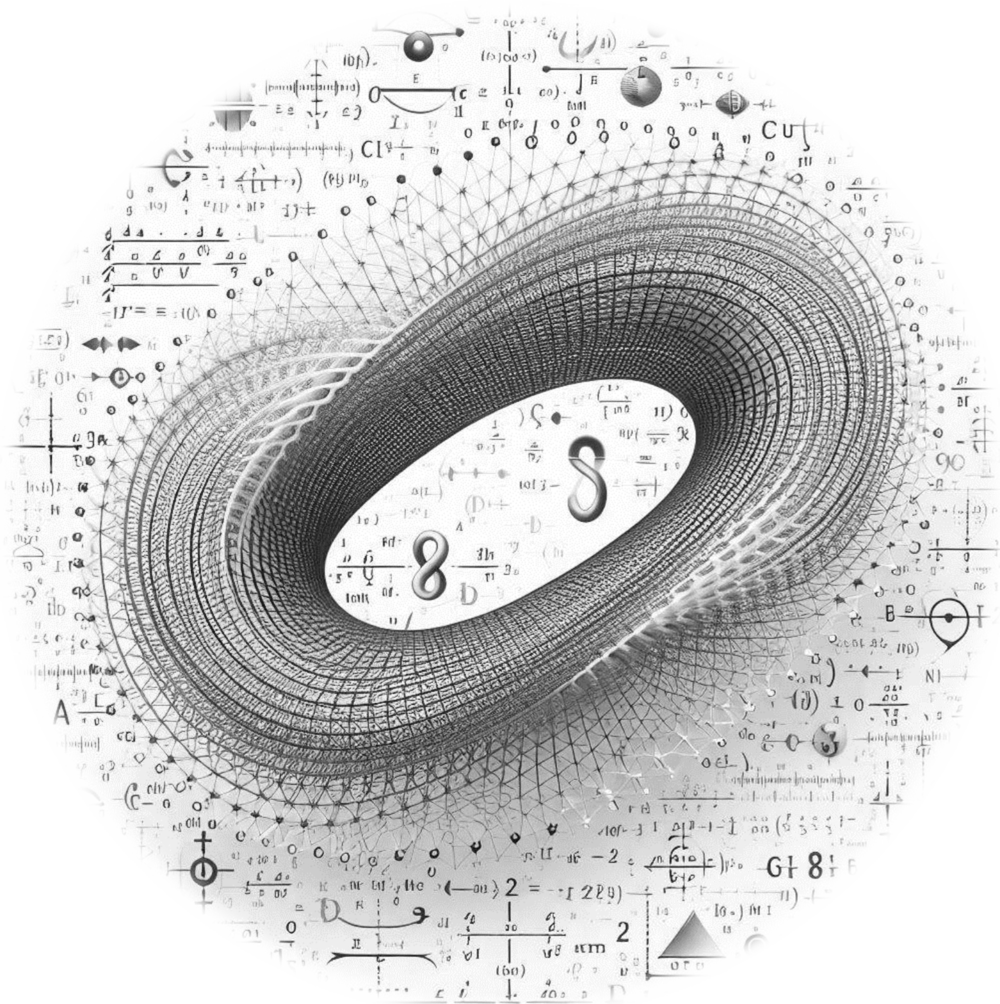
eSpyMath: Grade 6 Math Workbook

Contents

Chater 1. Fundamentals of Numbers and Operations



1-1. Place Value and Review (including decimals and large numbers)

1. $10+5=$ 2. $15-9=$ 3. $7\times3=$ 4. $28\div7=$	15 6 21 4

1. Natural Numbers:
- These are the numbers used for counting.
- Examples: 1, 2, 3, 4, ...

2. Whole Numbers:
- This set includes all natural numbers along with zero.
- Examples: 0, 1, 2, 3, 4, ...

3. Base Ten Number System:
- The standard system for denoting integer and non-integer numbers.
- Also known as the decimal system.
- Each position in a number represents a power of 10.

4. Place-Value Table:
- A table used to compare and visualize the magnitude of numbers.
- Helps in understanding the value of each digit in a number based on its position.

5.Place-Value Table Structure:
- Divides numbers into three main categories:

Millions			Thousands			Hundreds		

Example: Place-Value Table:

Millions			Thousands			Hundreds		
	1	5	6	2	2	3	9	6

Number Representation:

- The number 15,622,396 is written as:
- "Fifteen million six hundred twenty-two thousand three hundred ninety-six."

Exercise:

1. What is the place value of 5 in the number 8,592.763?	2. Write the number 7,046.209 in words.
3. What is the value of the digit 8 in the number 3,816.45?	4. How would you write 0.507 in words?
5. If you have a number 4,230.98, how do you write it in expanded form?	6. What is the decimal equivalent of the fraction 3/4?
7. Round the number 6,813.297 to the nearest tenth.	8. Convert the decimal 0.62 to a fraction in its simplest form.
9. If the digit in the tens place of a number is 3 times the digit in the ones place and the number is less than 100, what could the number be?	10. What is the product of the place values of 2 in the number 2,482.62?

Solutions:

1. What is the place value of 5 in the number 8,592.763? - First, identify the position of 5 in the given number. In 8,592.763, 5 is in the hundreds place. - So, the place value of 5 is hundreds or specifically 500.	**2. Write the number 7,046.209 in words.** - Break the number into parts: 7,046 and 209 after the decimal. - 7,046 in words is Seven thousand forty-six. - .209 after the decimal is two hundred nine thousandths. - Combined, the number in words is Seven thousand forty-six and two hundred nine thousandths.
3. What is the value of the digit 8 in the number 3,816.45? - The 8 is in the hundred's place. - Therefore, the value of 8 in the number is 800.	**4. How would you write 0.507 in words?** - 0.507 can be broken down as follows: 5 is in the tenths place, 0 is in the hundredths place, and 7 is in the thousandths place. - Thus, it is written as five hundred seven thousandths.
5. If you have a number 4,230.98, how do you write it in expanded form? - Break down each digit according to its place value: - 4,000 + 200 + 30 + 0.9 + 0.08 - So, 4,230.98 in expanded form is 4,000 + 200 + 30 + 9/10 + 8/100.	**6. What is the decimal equivalent of the fraction 3/4?** - Divide the numerator by the denominator: 3 ÷ 4 = 0.75. - So, the decimal equivalent of ¾ is 0.75.
7. Round the number 6,813.297 to the nearest tenth. - Look at the hundredth place, which is 9. - Since 9 is greater than 5, round the tenths place up from 2 to 3. - The number rounded to the nearest tenth is 6,813.3.	**8. Convert the decimal 0.62 to a fraction in its simplest form.** - 0.62 means 62 out of 100 or 62/100. - Divide both the numerator and denominator by their greatest common divisor, which is 2. - This gives us 31/50. - So, 0.62 as a fraction in its simplest form is 31/50.
9. If the digit in the tens place of a number is 3 times the digit in the ones place and the number is less than 100, what could the number be? - Let's assume the digit in the ones place is x. Then the digit in the tens place is 3x. - Since the number is less than 100, possible pairs can be (1, 3), (2, 6), or (3, 9). - Thus, the numbers could be 13, 26, or 39.	**10. What is the product of the place values of 2 in the number 2,482.62?** - The first 2 is in the thousands place, so its place value is 2,000. - The second 2 is in the ones place, so its place value is 2. - The third 2 is in the hundredths place, so its place value is 0.02. - The product of 2,000, 2, and 0.02 is 80. - Therefore, the product of the place values of 2 in 2,482.62 is 80.

1-2. Operations with Whole Numbers
(addition, subtraction, multiplication, division)

1. $(7+8)-3=$ 2. $9\times(2+1)=$ 3. $(12\div3)+4=$ 4. $7-(5-2)=$	12 27 8 4

1. Expanded Notation:

- Represents a number as the sum of each digit multiplied by its place value.
- Breaks down a number into parts, highlighting the value of each digit.
- Example: 574,789 in expanded notation is 500,000 + 70,000 + 4,000 + 700 + 80 + 9.

2. Standard Notation:

- The typical way of writing numbers.
- All the parts of the number are combined into a single numerical value.
- Example: 574,789.

3. Conversion between Notations:

- To Expanded Notation: Decompose the number according to its place values.
- To Standard Notation: Sum all the individual parts of the expanded notation to get the number in its conventional form.

Example: For the expanded notation 500,000 + 70,000 + 4,000 + 700 + 80 + 9

- $n=500,000+70,000+4,000+700+80+9=574,789$

Exercise:

1. What is 645 plus 289?	2. Subtract 432 from 789.
3. Multiply 123 by 3.	4. Divide 864 by 3.
5. Calculate the sum of 345, 678, and 912.	6. If you subtract 654 from 1,000, what is the result?
7. What is the product of 256 and 4?	8. How many times does 48 go into 4,320?
9. Find the difference between the product of 123 and 7 and the sum of 456 and 789.	10. A baker used 250 grams of flour to make a cake and 475 grams to make cookies. If he started with 1 kilogram (1,000 grams) of flour, how much flour does he have left?

Solutions:

1. What is 645 plus 289? - Add the numbers vertically: 645 + 298 943 - So, 645 plus 289 equals 934.	**2. Subtract 432 from 789.** Subtract the numbers vertically: 789 - 432 357 - Therefore, 789 minus 432 equals 357.
3. Multiply 123 by 3. - Multiply the numbers: 123 x 3 369 - So, 123 times 3 equals 369.	**4. Divide 864 by 3.** - Divide the numbers: - 864 ÷ 3 = 288 - Therefore, 864 divided by 3 equals 288.
5. Calculate the sum of 345, 678, and 912. - Add the numbers vertically: 345 678 + 912 1935 - So, the sum of 345, 678, and 912 is 1935.	**6. If you subtract 654 from 1,000, what is the result?** - Subtract the numbers: 1000 - 654 346 - The result of subtracting 654 from 1,000 is 346.
7. What is the product of 256 and 4? - Multiply the numbers: 256 x 4 1024 - Therefore, the product of 256 and 4 is 1024.	**8. How many times does 48 go into 4,320?** - Divide the numbers: 4320 ÷ 48 = 90 - So, 48 goes into 4,320 exactly 90 times.
9. Find the difference between the product of 123 and 7 and the sum of 456 and 789. - First, find the product of 123 and 7: - Then, find the sum of 456 and 789: - Subtract the sum from the product: - (123 x 7)+(456+789) = 861 – 1245 = -384 - The difference is -384, indicating that the sum is larger than the product by -384.	**10. A baker used 250 grams of flour to make a cake and 475 grams to make cookies. If he started with 1 kilogram (1,000 grams) of flour, how much flour does he have left?** - First, add the amount of flour used for the cake and cookies: 250 (cake) + 475 (cookies) = 725 grams - Subtract this from the starting amount: 1000 – 725 = 275 - The baker has 275 grams of flour left.

1-3. Divisibility Rules

1. $3+(4\times2)$ 2. $(15\div3)-2$ 3. $7-(2+3)$ 4. $(6\times2)\div3$	11 3 2 4

1. Divisibility:

- A number is said to be divisible by another if, when divided, there is no remainder.
- Example: A number is "divisible by 3" if dividing it by 3 leaves no remainder.

2. Evenly Divisible:

- This phrase emphasizes that the division results in no remainder.
- Example: 12 is evenly divisible by 4 (since $12\div4=3$ with no remainder).

Divisibility Rules (Brief Overview):

- 2: A number is divisible by 2 if it is even.
- 3: A number is divisible by 3 if the sum of its digits is divisible by 3.
- 5: A number is divisible by 5 if it ends in 0 or 5.
- 7: There are more complex rules for 7, but basic division is often used to check.

Divisibility Formula:

- For any integer a and a non-zero integer b : $a=b\times q+r$
- Where:
  - q is the quotient.
  - r is the remainder.
  - If r = 0 , then a is divisible by b .

Example: 85 divided by 7

- Calculation: $85\div7=12$ R 1
- Since there is a remainder of 1, 85 is not evenly divisible by 7.

Example: For $85\div7$

- $85=7\times12+1$
- Here, the remainder $r=1$, so 85 is not evenly divisible by 7.

1. Divisibility by 2:
- A number is divisible by 2 if its last digit is 0, 2, 4, 6, or 8.
- Example: 1006 is divisible by 2; 1007 is not.

2. Divisibility by 3:
- A number is divisible by 3 if the sum of its digits is divisible by 3.
- Example: For 2553, 2 + 5 + 5 + 3 = 15. Since 15 is divisible by 3, so is 2553.

3. Divisibility by 4:
- A number is divisible by 4 if the last two digits form a number divisible by 4.
- Example: For 291,583,791,283, the last two digits are 83. Since 83 is not divisible by 4, the whole number is not divisible by 4.

4. Divisibility by 5:
- A number is divisible by 5 if its last digit is 0 or 5.
- Example: 123450 is divisible by 5; 12345 is not.

5. Divisibility by 6:
- A number is divisible by 6 if it is divisible by both 2 and 3.
- Example: For 186, it is divisible by 2 (last digit is 6) and by 3 (1 + 8 + 6 = 15, which is divisible by 3). So, 186 is divisible by 6.

6. Divisibility by 9:
- A number is divisible by 9 if the sum of its digits is divisible by 9.
- Example: For 78,957, 7 + 8 + 9 + 5 + 7 = 36. Since 36 is divisible by 9, so is 78,957.

7. Divisibility by 10:
- A number is divisible by 10 if its last digit is 0.
- Example: 123450 is divisible by 10; 12345 is not.

Divisibility by 3:

Example: Is 2553 divisible by 3?

- Add the digits: $2 + 5 + 5 + 3 = 15$.
- Since 15 is divisible by 3, 2553 is divisible by 3.

Example: Is 2554 divisible by 3?

- Add the digits: 2 + 5 + 5 + 4 = 16.
- Since 16 is not divisible by 3, 2554 is not divisible by 3.

Divisibility by 4:

Example: Is 291,583,791,283 divisible by 4?

- Check the last two digits: 83.

- Since 83 is not divisible by 4, the whole number is not divisible by 4.

Divisibility by 6:

Example: Is 186 divisible by 6?

- Check if it is divisible by 2: Last digit is 6, so it is divisible by 2.
- Check if it is divisible by 3: 1 + 8 + 6 = 15, and 15 is divisible by 3.
- Since 186 is divisible by both 2 and 3, it is divisible by 6.

Divisibility by 9:

Example: Is 78,957 divisible by 9?

- Add the digits: 7 + 8 + 9 + 5 + 7 = 36.
- Add the digits of 36: 3 + 6 = 9.
- Since 9 is divisible by 9, 78,957 is divisible by 9.

Divisibility by 10:

Example: Which number, 12345 or 123450, is divisible by 10?

- 123450 is divisible by 10 because it ends in 0. 12345 is not because it ends in 5.

Exercise:

1. How can you tell if a number is divisible by 2?	2. What is the rule for a number to be divisible by 3?
3. Explain how to determine if a number is divisible by 4.	4. How can you tell if a number is divisible by 5?
5. What is the divisibility rule for 6?	6. Describe the rule for a number to be divisible by 9.
7. How can you determine if a number is divisible by 10?	8. What is the divisibility rule for 11?
9. Explain how to find out if a number is divisible by 12.	10. A number ends in 6 and when you sum all its digits, you get 9. Is it divisible by 3, 4, and 6? Justify your answer.

Solutions:

1. How can you tell if a number is divisible by 2? - A number is divisible by 2 if its last digit is even (0, 2, 4, 6, or 8).	**2. What is the rule for a number to be divisible by 3?** - A number is divisible by 3 if the sum of its digits is divisible by 3.
3. Explain how to determine if a number is divisible by 4. - A number is divisible by 4 if its last two digits form a number that is divisible by 4.	**4. How can you tell if a number is divisible by 5?** - A number is divisible by 5 if its last digit is either 0 or 5.
5. What is the divisibility rule for 6? - A number is divisible by 6 if it is divisible by both 2 and 3.	**6. Describe the rule for a number to be divisible by 9.** - A number is divisible by 9 if the sum of its digits is divisible by 9.
7. How can you determine if a number is divisible by 10? - A number is divisible by 10 if its last digit is 0.	**8. What is the divisibility rule for 11?** - A number is divisible by 11 if the difference between the sum of the digits in the odd positions and the sum of the digits in the even positions is either 0 or a multiple of 11. - For example, for the number 121, the sums are 1+1=2 and 2, and the difference is 2−2=0, which is divisible by 11
9. Explain how to find out if a number is divisible by 12. - A number is divisible by 12 if it meets the divisibility rules for both 3 and 4.	**10. A number ends in 6 and when you sum all its digits, you get 9. Is it divisible by 3, 4, and 6? Justify your answer.** - For divisibility by 3. Since the sum of its digits is 9, which is divisible by 3, the number is divisible by 3. - For divisibility by 4. A number ending in 6 cannot be judged for divisibility by 4 without knowing its last two digits. - However, since we only know it ends in 6, we cannot confirm divisibility by 4 based on the given information. - For divisibility by 6. Since the number is divisible by 3 (from the sum of its digits) and ends in an even number (6), it is also divisible by 2. Therefore, it meets the criteria for divisibility by 6. - Hence, the number is definitely divisible by 3 and 6. Divisibility by 4 cannot be determined with the given information.

1-4. Prime Factorization and Prime Numbers

1. $2\times(3+5)$ 2. $10\div(2+3)$ 3. $(4\times3)-5$ 4. $9+(12\div3)$	16 2 7 13

1. Prime Numbers:

- A prime number is a whole number greater than 1 that has exactly two distinct factors: 1 and itself.
- Example: 7 is a prime number because its only factors are 1 and 7.
- Note: The number 1 is **not** considered a prime number.

2. Composite Numbers:

- A composite number is a whole number greater than 1 that has more than two distinct factors.
- Example: 4 is a composite number because it has three factors: 1, 2, and 4.
- Every whole number greater than 1 is either a prime number or a composite number.

3. Prime Factorization:

- Prime factorization is the process of expressing a number as the product of its prime factors.
- Example: The prime factorization of 45 is $3\times3\times5$ or $3^2\times5$.

1. Identifying Prime Numbers:

- To check if a number n is prime, ensure it has no divisors other than 1 and n itself.
- Example: 11 is prime because its only divisors are 1 and 11.

2. Identifying Composite Numbers:

- A number n is composite if it can be divided evenly by at least one positive integer other than 1 and n.
- Example: 12 is composite because it has factors 1, 2, 3, 4, 6, and 12.

3. Prime Factorization Process:

- Break down the number into its prime factors.
- Example: Factorizing 45: $45=3\times3\times5=3^2\times5$
- Non-prime factorization like $45=9\times5$ is not correct because 9 is not a prime number.

4. Combining Prime Factors into Exponents:

- When a prime factor appears multiple times, use exponents to simplify.
- Example: $45 = 3 \times 3 \times 5$ can be written as $3^2 \times 5$.

1. Multiples:

- When a whole number is multiplied by another whole number, the results are called multiples.
- Example: Multiples of 3 include 3, 6, 9, 12, ...

2. Finding Multiples:

- To find the first n multiples of a number, multiply the number by each of the first n whole numbers.
- Example: The first four multiples of 3 are calculated as:
- $3 \times 1 = 3$
- $3 \times 2 = 6$
- $3 \times 3 = 9$
- $3 \times 4 = 12$

3. Divisibility:

- Multiples of a number are always divisible by that number.
- Example: All multiples of 3 (such as 3, 6, 9, 12) can be divided evenly by 3.

1. Factors:

- Factors are numbers that are multiplied together to get a product.
- Example: In $4 \times 5 \times 3 = 60$, the numbers 4, 5, and 3 are factors of 60.

2. Product:

- The result of multiplying factors together.
- Example: The product of $4 \times 5 \times 3$ is 60.

3. Division Terms:

- Dividend: The number being divided.
- Divisor: The number by which the dividend is divided.
- Quotient: The result of the division.
- Example: In $25 \div 5 = 5$, 25 is the dividend, 5 is the divisor, and 5 is the quotient.

Exercise

1. What is a prime number?	2. Is the number 7 a prime number?
3. Find the prime factorization of 36.	4. What are the prime factors of 50?
5. How can you tell if a number is not a prime number?	6. Identify all the prime numbers between 10 and 20.
7. Perform prime factorization of 84.	8. What is the smallest prime number?
9. Can the number 1 be considered a prime number? Why or why not?	10. What are the prime factors of 97?

Solutions:

1. What is a prime number? - A prime number is a whole number greater than 1 that has exactly two distinct positive divisors: 1 and itself. - This means it can only be divided evenly by 1 and the number itself without leaving a remainder.	**2. Is the number 7 a prime number?** - Yes, 7 is a prime number because it can only be divided evenly by 1 and 7.
3. Find the prime factorization of 36. - Start by dividing 36 by the smallest prime number, 2: 36 ÷ 2 = 18 - Then, divide 18 by 2 again: 18 ÷ 2 = 9 - Since 9 cannot be divided evenly by 2, move to the next prime number, 3: 9 ÷ 3 = 3 - Finally, divide 3 by itself: 3 ÷ 3 = 1 - The prime factorization of 36 is 2 × 2 × 3 × 3 or. $2^2 \times 3^2$.	**4. What are the prime factors of 50?** - Start by dividing 50 by the smallest prime number, 2: 50 ÷ 2 = 25 - 25 cannot be divided by 2, so move to the next prime number, which is 3. Since 25 is not divisible by 3, move to 5: 25 ÷ 5 = 5 - Finally, divide 5 by itself: 5 ÷ 5 = 1 - The prime factors of 50 are 2 and 5, and its prime factorization is 2×5^2.
5. How can you tell if a number is not a prime number? - A number is not a prime number if it can be divided evenly by a whole number other than 1 and itself. - If a number has more than two distinct positive divisors, it is not a prime number.	**6. Identify all the prime numbers between 10 and 20.** - The prime numbers between 10 and 20 are 11, 13, 17, and 19. - These numbers cannot be divided evenly by any number other than 1 and themselves.
7. Perform prime factorization of 84. - Start dividing 84 by the smallest prime number, 2: 84 ÷ 2 = 42 - Continue with 2: 42 ÷ 2 = 21 - Since 21 is not divisible by 2, move to the next prime number, 3: 21 ÷ 3 = 7 - Finally, 7 is a prime number and can only be divided by itself: 7 ÷ 7 = 1 - The prime factorization of 84 is $2^2 \times 3 \times 7$.	**8. What is the smallest prime number?** - The smallest prime number is 2. - It is also the only even prime number because every other even number can be divided by 2.
9. Can the number 1 be considered a prime number? Why or why not? - No, 1 cannot be considered a prime number. - By definition, a prime number must have exactly two distinct positive divisors: 1 and itself. - Since 1 only has one distinct positive divisor (itself), it does not meet the criteria for being a prime number.	**10. What are the prime factors of 97?** - 97 is itself a prime number because it cannot be divided evenly by any other number except 1 and 97. - Therefore, its only prime factor is 97 itself.

1-5. Powers and Exponents (including Powers of 10)

1. $(5+3)\times(2+1)$	24
2. $18\div(3+3)+4$	7
3. $(6-2)\times(5-3)$	8
4. $(8\div2)+(10-7)$	7

1. Exponents:
- Exponents represent repeated multiplication of a number by itself.
- Notation: a^n, where a is the base and n is the exponent.
- Example: 3^6 means $3\times3\times3\times3\times3\times3$.

$$a^n = \underbrace{a\times a\times\ldots\times a}_{n \text{ times}}$$

2. Base:
- The number that is being multiplied.
- Example: In 3^6, the base is 3.

3. Exponent:
- The number of times the base is multiplied by itself.
- Example: In 3^6, the exponent is 6.

Tip 1: If the exponent is 0, the result is always 1.

- $2^0 = 1$, $29^0 = 1$, $1^0 = 1$

Tip 2: If the exponent is 1, the result is always the base number.

- $2^1 = 2$, $29^1 = 29$, $1^1 = 1$, $0^1 = 0$

Tip 3: If the base is 0, the result is always 0 (except for $ 0^0 $).

- $0^2 = 0$, $0^{1000} = 0$

Tip 4: If the base is 1, the result is always 1.

- $1^2 = 1$, $1^{1000} = 1$

Tip 5: 0^0 is an **indeterminate** form and doesn't have a defined value in basic arithmetic.

Exercise

1. What is an exponent in a mathematical expression?	2. Calculate 5^3.
3. What is 10^4?	4. How would you write one million in powers of 10?
5. If $2^x = 64$, what is the value of x?	6. Simplify $3^2 \times 3^4$.
7. What is the value of 10^{-3}?	8. Divide 2^8 by 2^3.
9. If $5^2 \times 5^x = 5^7$, what is the value of x?	10. What is 2^0?

Solutions:

1. What is an exponent in a mathematical expression?

- An exponent in a mathematical expression indicates how many times a number, known as the base, is multiplied by itself.
- For example, in 2^3, 2 is the base and 3 is the exponent, meaning. $2 \times 2 \times 2 = 8$.

2. Calculate 5^3.

- To calculate 5^3, multiply 5 by itself three times: $5 \times 5 \times 5 = 125$.
- Therefore, $5^3 = 125$.

3. What is 10^4?

- 10^4 means 10 multiplied by itself 4 times: $10 \times 10 \times 10 \times 10 = 10{,}000$.
- So, $10^4 = 10{,}000$.

4. How would you write one million in powers of 10?

- One million is 1 followed by six zeros, which can be written as 10^6.
- Therefore, one million in powers of 10 is 10^6.

5. If $2^x = 64$, what is the value of x?

- To find x, determine how many times 2 must be multiplied by itself to equal 64:
  - $2 \times 2 = 4$, $2^2 = 4$
  - $2 \times 2 \times 2 = 8$, $2^3 = 8$
  - $2 \times 2 \times 2 \times 2 = 16$, $2^4 = 16$
  - $2 \times 2 \times 2 \times 2 \times 2 = 32$, $2^5 = 32$
  - $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$, $2^6 = 64$
- Therefore, $x = 6$.

6. Simplify $3^2 \times 3^4$.

- When multiplying powers with the same base, add the exponents: $3^2 \times 3^4 = 3^{2+4} = 3^6$.
- Calculate $3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$.
- So, $3^2 \times 3^4 = 3^6 = 729$.

7. What is the value of 10^{-3}?

- A negative exponent means the reciprocal of the base raised to the absolute value of the exponent: $10^{-3} = \frac{1}{10^3} = \frac{1}{1000}$.
- Therefore, $10^{-3} = 0.001$.

8. Divide 2^8 by 2^3.

- When dividing powers with the same base, subtract the exponents: $2^8 \div 2^3 = 2^{8-3} = 2^5$.
- Calculate $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$.
- So, 2^8 divided by 2^3 equals 32.

9. If $5^2 \times 5^x = 5^7$, what is the value of x?

- According to the laws of exponents, when you multiply powers with the same base, you add the exponents: $5^2 \times 5^x = 5^{2+x}$.
- Since $5^{2+x} = 5^7$, it follows that $2 + x = 7$.
- Solving for x, $x = 7 - 2 = 5$.
- Therefore, the value of x is 5.

10. What is 2^0?

- Any number raised to the power of 0 equals 1 (excluding 0^0, which is undefined).
- Therefore, $2^0 = 1$.

1-6. Square Numbers and Square Roots

1. $7+(9-5)\times 2=$ 2. $(12+8)\div(2+2)=$ 3. $5\times(2+3)-10=$ 4. $(15-5)\div(2+3)=$	15 5 15 2

Exercise

1. What is a square number?	2. Calculate the square of 8.
3. What is the square root of 36?	4. Is 49 a square number? If yes, what is its square root?
5. How can you tell if a number is a square number by looking at it?	6. What are the first five square numbers?
7. Find the square root of 81.	8. Calculate the square of 15.
9. If $x^2=100$, what are the possible values of x?	10. What is the square root of 0.25?

Solutions:

1. What is a square number? - A square number is a number that can be expressed as the product of an integer with itself. - For example, 4 is a square number because it can be written as 2×2 .	**2. Calculate the square of 8.** - The square of 8 is $8 \times 8 = 64$. - Therefore, the square of 8 is 64.
3. What is the square root of 36? - The square root of 36 is the number that, when multiplied by itself, gives 36. - This number is 6, because $6 \times 6 = 36$. - So, the square root of 36 is 6.	**4. Is 49 a square number? If yes, what is its square root?** - Yes, 49 is a square number because it can be expressed as 7×7 . - The square root of 49 is 7.
5. How can you tell if a number is a square number by looking at it? - It can be challenging to tell if a large number is a square number just by looking at it, but some hints may include: - The last digit: Square numbers end in 0, 1, 4, 5, 6, or 9 in the decimal system. - The number of digits: A square number often results from squaring a small number, so familiarizing yourself with the squares of numbers 1-12 can help identify larger square numbers. - However, the most reliable method is to calculate the square root and see if it's an integer.	**6. What are the first five square numbers?** - The first five square numbers are the squares of the first five positive integers: ○ $1^2 = 1$ ○ $2^2 = 4$ ○ $3^2 = 9$ ○ $4^2 = 16$ ○ $5^2 = 25$ - So, the first five square numbers are 1, 4, 9, 16, and 25.
7. Find the square root of 81. - The square root of 81 is the number that, when multiplied by itself, equals 81. - This number is 9 because $9 \times 9 = 81$. - Therefore, the square root of 81 is 9.	**8. Calculate the square of 15.** - The square of 15 is $15 \times 15 = 225$. - So, the square of 15 is 225.
9. If $x^2 = 100$, what are the possible values of x ? - The equation $x^2 = 100$ has two solutions because two numbers multiplied by themselves can result in 100: ○ $10 \times 10 = 100$ ○ $(-10) \times (-10) = 100$ - Therefore, the possible values of x are 10 and -10.	**10. What is the square root of 0.25?** - The square root of 0.25 is the number that, when multiplied by itself, gives 0.25. - This number is 0.5 because $0.5 \times 0.5 = 0.25$. - So, the square root of 0.25 is 0.5.

1-7. Greatest Common Factor (GCF)

1. $(3+4)\times(5-2)$ 2. $(9\div3)+(8-2)$ 3. $4\times(7-3)$ 4. $(18\div(2+1))\times2$	21 9 16 12

1. Factor Tree:

- A diagram is used to break down a number into its prime factors.
- The process involves dividing the number by its smallest prime factor repeatedly until all factors are prime numbers.

2. Prime Factorization:

- The representation of a number as a product of its prime factors.
- Example: The prime factorization of 252 is $2^2\times3^2\times7$.

Steps to Create a Factor Tree:

1. Start with the number at the top.
2. Divide the number by the smallest prime factor.
3. Write the quotient below and repeat the process with the quotient.
4. Continue until all resulting numbers are prime.

Example: Prime Factorization of 252

- Start with 252: 252 is even, divide by 2: $252\div2=126$
- Continue with 126: 126 is even, divide by 2: $126\div2=63$
- Continue with 63: 63 is divisible by 3: $63\div3=21$
- Continue with 21: 21 is divisible by 3: $21\div3=7$
- 7 is a prime number.
- So, the prime factorization of 252 is: $252=2\times2\times3\times3\times7=2^2\times3^2\times7$

Example: Prime Factorization of 80

- Start with 80: 80 is even, divide by 2: $80\div2=40$
- Continue with 40: 40 is even, divide by 2: $40\div2=20$
- Continue with 20: 20 is even, divide by 2: $20\div2=10$
- Continue with 10: 10 is even, divide by 2: $10\div2=5$
- 5 is a prime number.
- So, the prime factorization of 80 is: $80=2\times2\times2\times2\times5=2^4\times5$

1. Greatest Common Factor (GCF):

- The largest number is a common factor of two or more numbers.
- Abbreviated as GCF.

2. Steps to Find the GCF:

- List all the factors of each number.
- Identify the common factors of the numbers.
- Choose the largest factor that is common to both numbers.

3. Prime Factorization Method:

- Factor each number into its prime factors.
- Identify the common prime factors.
- Multiply the common prime factors to find the GCF.
- Example: Find the GCF of 36 and 42.
    - Prime factorization of 36: $2^2 \times 3^2$
    - Prime factorization of 42: $2 \times 3 \times 7$
    - Common prime factors: 2, 3
- GCF: $2 \times 3 = 6$

Example: Use the steps to find the GCF of 48 and 60:

- Factors of 48: $\{1,2,3,4,6,8,12,16,24,48\}$
- Factors of 60: $\{1,2,3,4,5,6,10,12,15,20,30,60\}$
- Common factors: $\{1,2,3,4,6,12\}$
- GCF: 12

Example: Finding the GCF of 36 and 42:

- Step 1: List the factors of each number.
    - Factors of 36: $\{1,2,3,4,6,9,12,18,36\}$
    - Factors of 42: $\{1,2,3,6,7,14,21,42\}$
- Step 2: Identify the common factors.
    - Common factors of 36 and 42: $\{1,2,3,6\}$
- Step 3: Choose the largest common factor.
    - The largest common factor is 6.
- So, the GCF of 36 and 42 is 6.

Exercise

1. What is the Greatest Common Factor (GCF)?	2. Find the GCF of 18 and 24.
3. How do you calculate the GCF of three numbers, like 20, 30, and 40?	4. What is the GCF of 48 and 180?
5. Is it possible for two numbers to have a GCF of 1? If so, what does that indicate about the numbers?	6. Determine the GCF of 81 and 108.
7. How do you find the GCF of two prime numbers?	8. What is the GCF of 14, 49, and 63?
9. Explain why the GCF of any number and itself is the number itself.	10. Can the GCF of a set of numbers be larger than the smallest number in the set? Why or why not?

Solutions:

1. What is the Greatest Common Factor (GCF)?

- The Greatest Common Factor (GCF) of two or more numbers is the largest number that divides evenly into all the numbers without leaving a remainder.
- It's also known as the greatest common divisor (GCD).

2. Find the GCF of 18 and 24.

- First, list the factors of each number:
- Factors of 18. 1, 2, 3, 6, 9, 18
- Factors of 24. 1, 2, 3, 4, 6, 8, 12, 24
- The common factors are 1, 2, 3, and 6.
- The largest of these is 6.
- Therefore, the GCF of 18 and 24 is 2x3=6.

2	18	24
3	9	12
	3	4

3. How do you calculate the GCF of three numbers, like 20, 30, and 40?

- List the factors of each number:
- Factors of 20: 1, 2, 4, 5, 10, 20
- Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30
- Factors of 40: 1, 2, 4, 5, 8, 10, 20, 40
- Identify the common factors: 1, 2, 5, and 10.
- The largest common factor is 10.
- So, the GCF of 20, 30, and 40 is 10.

10	20	30	40
	2	3	4

4. What is the GCF of 48 and 180?

- Factors of 48. 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
- Factors of 180: 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 30, 36, 45, 60, 90, 180
- The common factors are 1, 2, 3, 4, 6, and 12.
- The largest of these is 12.
- Therefore, the GCF of 48 and 180 is 4x3=12.

4	48	180
3	12	45
	4	15

5. Is it possible for two numbers to have a GCF of 1? If so, what does that indicate about the numbers?

- Yes, it is possible for two numbers to have a GCF of 1.
- This indicates that the numbers are coprime (or relatively prime), meaning they have no common positive factors other than 1.
- The numbers do not necessarily have to be prime themselves.

6. Determine the GCF of 81 and 108.

- Factors of 81. 1, 3, 9, 27, 81
- Factors of 108. 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108
- The common factors are 1, 3, 9, and 27.
- The largest common factor is 27.
- So, the GCF of 81 and 108 is 9x3=27.

9	81	108
3	9	12
	3	4

7. How do you find the GCF of two prime numbers?

- Since prime numbers have no other divisors besides 1 and themselves, the GCF of two distinct prime numbers is always 1.
- This is because the only common factor they have is 1.

8. What is the GCF of 14, 49, and 63?

- Factors of 14. 1, 2, 7, 14
- Factors of 49: 1, 7, 49
- Factors of 63. 1, 3, 7, 9, 21, 63
- The common factor among all three is 7.
- Therefore, the GCF of 14, 49, and 63 is 7.

	7 | 14 49 63 2 7 9
9. Explain why the GCF of any number and itself is the number itself. - The GCF of a number and itself is the number itself because the greatest common factor is defined as the largest factor that divides both numbers. - Since the number divides itself exactly, it is the largest factor they have in common.	**10. Can the GCF of a set of numbers be larger than the smallest number in the set? Why or why not?** - No, the GCF of a set of numbers cannot be larger than the smallest number in the set. - This is because the GCF is a factor that all numbers in the set share, and a factor of a number cannot be larger than the number itself. - Therefore, the GCF must be equal to or less than the smallest number in the set.

1-8. Least Common Multiple (LCM)

1. $(6+2)-(3\times2)$	2
2. $(10\div2)+(4\times2)$	13
3. $8-(4\div(2+2))$	7
4. $(7+5)\times(3-1)$	24

1. Least Common Multiple (LCM):

- The smallest number is a multiple of two or more numbers.
- Abbreviated as LCM.

2. Steps to Find the LCM:

- List several multiples of each number.
- Identify the smallest multiple that is common to both numbers.

3. Prime Factorization Method:

- Find the prime factorization of each number.
- Multiply the highest power of each prime factor present in the factorization of both numbers.
- Example: Find the LCM of 14 and 26.
    - Prime factorization of 14: 2×7
    - Prime factorization of 26: 2×13
    - LCM: $2^1\times7^1\times13^1=182$

Example: Finding the LCM of 14 and 26:

- Step 1: List the multiples of each number.
    - Multiples of 14: $\{14,28,42,56,70,84,98,112,126,140,154,168,182,196,\ldots\}$
    - Multiples of 26: $\{26,52,78,104,130,156,182,208,\ldots\}$
- Step 2: Identify the smallest common multiple.
    - Common multiples of 14 and 26: $\{182,\ldots\}$
- The smallest common multiple is 182.
- So, the LCM of 14 and 26 is 182.

Example: Use the steps to find the LCM of 12 and 15:

- Multiples of 12: $\{12,24,36,48,60,72,84,96,\ldots\}$
- Multiples of 15: $\{15,30,45,60,75,90,105,\ldots\}$
- Common multiples: $\{60,\ldots\}$
- LCM: 60

Exercise

1. What is the Least Common Multiple (LCM)?	2. Find the LCM of 4 and 5.
3. How do you calculate the LCM of three numbers, like 3, 4, and 5?	4. What is the LCM of 6 and 8?
5. Is it possible for the LCM of two numbers to be one of the numbers?	6. Determine the LCM of 12 and 15.
7. How do you find the LCM if the numbers are prime?	8. What is the LCM of 10, 20, and 30?
9. Can the LCM of a set of numbers be smaller than the largest number in the set? Why or why not?	10. Determine the LCM of 7 and 14.

Solutions:

1. What is the Least Common Multiple (LCM)?

- The Least Common Multiple (LCM) of two or more numbers is the smallest number that is a multiple of all the numbers in the set.
- It's the lowest number that all the original numbers can divide into without leaving a remainder.

2. Find the LCM of 4 and 5.

- List some multiples of 4. 4, 8, 12, 16, 20, ...
- List some multiples of 5. 5, 10, 15, 20, 25, ...
- The smallest common multiple in both lists is 20.
- Therefore, the LCM of 4 and 5 is 1x4x5=20.

1	4	5
	4	5

3. How do you calculate the LCM of three numbers, like 3, 4, and 5?

- List some multiples of each number:
- Multiples of 3. 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, ...
- Multiples of 4. 4, 8, 12, 16, 20, 24, 28, 32, ...
- Multiples of 5. 5, 10, 15, 20, 25, 30, 35, 40, ...
- The smallest number common in all lists is 60.
- So, the LCM of 3, 4, and 5 is 60.

4. What is the LCM of 6 and 8?

- Multiples of 6. 6, 12, 18, 24, 30, 36, ...
- Multiples of 8. 8, 16, 24, 32, 40, ...
- The smallest common multiple is 24.
- Therefore, the LCM of 6 and 8 is 2x3x4=24.

2	6	8
	3	4

5. Is it possible for the LCM of two numbers to be one of the numbers?

- Yes, it is possible for the LCM of two numbers to be one of the numbers, especially if one number is a multiple of the other.
- For example, the LCM of 2 and 4 is 4, since 4 is a multiple of 2 and obviously of itself.

6. Determine the LCM of 12 and 15.

- Multiples of 12. 12, 24, 36, 48, 60, 72, ...
- Multiples of 15. 15, 30, 45, 60, 75, ...
- The smallest number common to both lists is 60.
- So, the LCM of 12 and 15 is 3x4x5=60.

3	12	15
	4	5

7. How do you find the LCM if the numbers are prime?

- When both numbers are prime, the LCM is simply the product of those numbers since prime numbers have no common divisors other than 1.
- For example, the LCM of 3 and 5, both primes, is $3 \times 5 = 15$.

8. What is the LCM of 10, 20, and 30?

- Multiples of 10: 10, 20, 30, 40, 50, 60, ...
- Multiples of 20: 20, 40, 60, 80, 100, ...
- Multiples of 30: 30, 60, 90, 120, ...
- The smallest number common to all lists is 60.
- Therefore, the LCM of 10, 20, and 30 is 10x1x2x3=60.

10	10	20	30
	1	2	3

9. Can the LCM of a set of numbers be smaller than the largest number in the set? Why or why not?

- No, the LCM of a set of numbers cannot be smaller than the largest number in the set because the LCM is a multiple that all the

10. Determine the LCM of 7 and 14.

- Multiples of 7. 7, 14, 21, 28, 35, ...
- Multiples of 14. 14, 28, 42, 56, ...
- The smallest number common to both lists is 14.

original numbers can divide into without leaving a remainder. - Since the LCM has to be divisible by the largest number, it cannot be smaller than that number.	- Since 14 is a multiple of 7, the LCM of 7 and 14 is 7x1x2=14. 7 | 7 14 1 2

1-9. Order of Operations (PEMDAS/BODMAS)

1. $(5+7)\times 2-(3+4)$ 2. $12\div(6\div 2)+7$ 3. $(8-2)\times(4\div 2)$ 4. $15-(3+2)\times(4-2)$	17 11 12 5

Exercise

1. What does PEMDAS stand for?	2. Simplify the expression $3+6\times 2$.
3. Calculate the value of $(4+3)\times 2$.	4. What is the result of $8\div 2(2+2)$?
5. Simplify $16-3^2+1$.	6. How do you evaluate $(5+3^2)\div 2$?
7. What is the outcome of $4^2-(2+6)\times 3$?	8. Evaluate $100\div(5\times 2)^2$.
9. What does the expression $3+6\times(5+4)\div 3-7$ equal?	10. Simplify $(2^3+2)\times(5-3^2)$.

Solutions:

1. What does PEMDAS stand for?

- PEMDAS stands for Parentheses, Exponents, Multiplication and Division (from left to right), and Addition and Subtraction (from left to right).
- It's a mnemonic for remembering the order of operations in mathematics.

2. Simplify the expression $3+6\times 2$.

- According to PEMDAS, you should perform multiplication before addition:
- First, multiply $6\times 2=12$.
- Then, add 3 to the result: $3+12=15$.
- Therefore, $3+6\times 2=15$.

3. Calculate the value of $(4+3)\times 2$.

- Following PEMDAS, calculate the expression inside the parentheses first:
- $4+3=7$.
- Then, multiply the result by 2: $7\times 2=14$.
- So, $(4+3)\times 2=14$.

4. What is the result of $8\div 2(2+2)$?

- First, solve the expression inside the parentheses: $2+2=4$.
- Then, according to PEMDAS, you have $8\div 2\times 4$.
- Multiplication and division are performed from left to right:
- $8\div 2=4$, then $4\times 4=16$.
- Therefore, $8\div 2(2+2)=16$.

5. Simplify $16-3^2+1$.

- Following the order of operations, calculate the exponent first: $3^2=9$.
- Then perform subtraction and addition from left to right:
- $16-9=7$, $7+1=8$.
- Therefore, $16-3^2+1=8$.

6. How do you evaluate $(5+3^2)\div 2$?

- First, calculate the exponent inside the parentheses: $3^2=9$.
- Then, add 5: $5+9=14$.
- Finally, divide by 2: $14\div 2=7$.
- So, $(5+3^2)\div 2=7$.

7. What is the outcome of $4^2-(2+6)\times 3$?

- First, solve the operation inside the parentheses: $2+6=8$.
- Then, calculate the exponent: $4^2=16$.
- Finally, multiply and subtract in order: $8\times 3=24$, then $16-24=-8$.
- Therefore, $4^2-(2+6)\times 3=-8$.

8. Evaluate $100\div(5\times 2)^2$.

- First, calculate the operation inside the parentheses: $5\times 2=10$.
- Then, square the result: $10^2=100$.
- Finally, divide 100 by 100: $100\div 100=1$.
- So, $100\div(5\times 2)^2=1$.

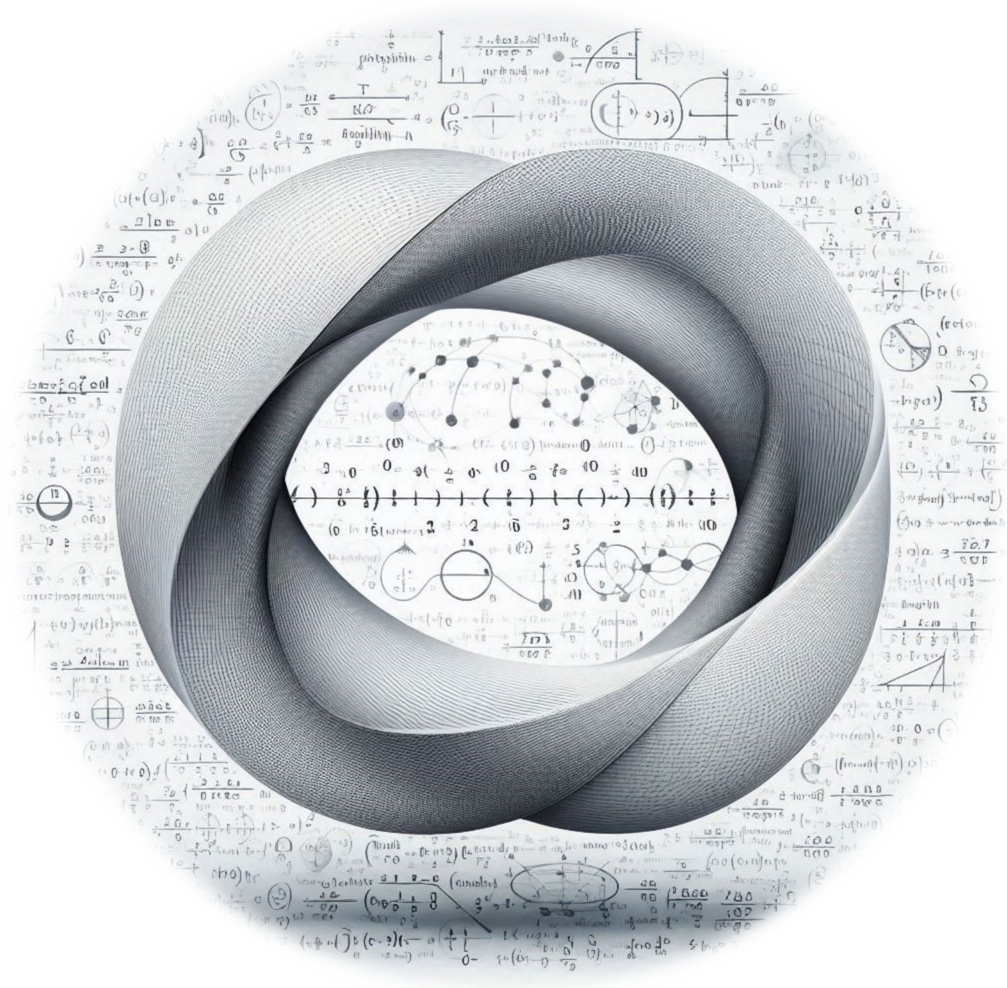
9. What does the expression $3+6\times(5+4)\div 3-7$ equal?

- First, solve the parentheses: $5+4=9$.
- Then, follow PEMDAS for the rest of the expression: $6\times 9=54$, $54\div 3=18$.
- Next, perform addition and subtraction from left to right: $3+18=21$, $21-7=14$.
- Therefore, $3+6\times(5+4)\div 3-7=14$.

10. Simplify $(2^3+2)\times(5-3^2)$.

- First, calculate the exponents: $2^3=8$ and $3^2=9$.
- Then, perform the operations inside the parentheses: $8+2=10$ and $5-9=-4$.
- Finally, multiply the results: $10\times -4=-40$.
- So, $(2^3+2)\times(5-3^2)=-40$.

Chater 7. Advanced Problem Solving and Mathematical Reasoning



7-1. Solving Multi-Step Word Problems using Algebra

1. $7\times[(4+6)-3]+[(2\times2)+1]\times4$ 2. $[18\div(9-6)]+(5\times2)-[(4-2)\times3]$ 3. $[(10-5)\times(2+3)]-(8\div4)+7$ 4. $8+(12\div3)\times[(6-2)+5]$	69 10 30 44

Exercise

1. What are multi-step word problems?	2. How do you solve a multi-step word problem using algebra?
3. What is the importance of defining variables in solving algebraic word problems?	4. How can drawing a diagram help in solving a multi-step word problem?
5. What are some common strategies for solving multi-step algebraic word problems?	6. How do you check your solution to a multi-step word problem?
7. Can multi-step word problems involve more than one variable?	8. How does understanding the context of a problem help in solving it?
9. Why is it important to simplify equations in multi-step problems?	10. What role does estimation play in solving multi-step word problems?

Solutions:

1. What are multi-step word problems? - Multi-step word problems require you to perform more than one mathematical operation to find the solution. - These problems involve a series of steps, including addition, subtraction, multiplication, division, or algebraic expressions to solve.	**2. How do you solve a multi-step word problem using algebra?** - To solve a multi-step word problem using algebra, follow these steps: - Read the problem carefully to understand what is being asked. - Identify and define the variables for unknown quantities. - Write down the equations based on the relationships and information given in the problem. - Solve the equations using appropriate algebraic methods, such as simplification, substitution, or elimination. - Check your solution by substituting the values back into the original equations.
3. What is the importance of defining variables in solving algebraic word problems? - Defining variables is crucial in solving algebraic word problems because it helps you translate the words into mathematical expressions and equations. - Variables represent the unknown quantities you need to find, making it easier to organize and solve the problem.	**4. How can drawing a diagram help in solving a multi-step word problem?** - Drawing a diagram can help visualize the problem, making it easier to understand the relationships between different parts of the problem. - It can also aid in identifying the variables, setting up the equations, and keeping track of the information provided and what needs to be found.
5. What are some common strategies for solving multi-step algebraic word problems? - Some common strategies include: - Breaking the problem down into smaller, manageable steps. - Using diagrams or charts to organize information. - Writing equations to represent the relationships described in the problem. - Checking the solution by substituting it back into the original problem.	**6. How do you check your solution to a multi-step word problem?** - To check your solution to a multi-step word problem, substitute the values you found back into the original equations or the problem statement to see if they satisfy all the given conditions and accurately answer the question posed.
7. Can multi-step word problems involve more than one variable? - Yes, multi-step word problems can involve more than one variable. - When this happens, you may need to set up a system of equations to solve for the unknown quantities.	**8. How does understanding the context of a problem help in solving it?** - Understanding the context of a problem helps you make sense of the information given, identify what the problem is asking for, and determine the appropriate mathematical operations or equations to use. It ensures that your solution is relevant and accurately addresses the question.

9. Why is it important to simplify equations in multi-step problems?	**10. What role does estimation play in solving multi-step word problems?**
- Simplifying equations in multi-step problems makes them easier to solve by reducing complexity, combining like terms, and eliminating unnecessary parts. - This can lead to clearer, more straightforward paths to finding the solution.	- Estimation can help you quickly assess whether your solution is reasonable by providing an approximate answer to compare with. - It can also assist in checking the plausibility of your results and ensuring that the calculations are headed in the right direction.

7-2. Applying Mathematical Reasoning

1. $2\times[(3+6)\times 2]-[(2\times 3)+4]$ 2. $[16\div(9-5)]+(5\times 2)-[(3+2)\times 2]$ 3. $[(10-6)\times(5+1)]-(8\div 2)+8$ 4. $7+(12\div 3)\times[(6-2)+4]-2$	26 4 28 37

Applying Mathematical Reasoning

1. Understanding the Problem:
- Carefully read the problem to understand what is being asked.
- Identify the known variables and the unknowns.
- Determine the conditions and constraints provided in the problem.

2. Strategizing the Approach:
- Choose an appropriate method to solve the problem (e.g., algebraic manipulation, geometric visualization, or numerical calculation).
- Break the problem into smaller, manageable parts if necessary.
- Consider if there are any patterns or relationships that can simplify the problem.

3. Executing the Plan:
- Carry out the chosen strategy methodically.
- Ensure each step logically follows from the previous one.
- Perform accurate calculations and algebraic manipulations.

4. Reviewing the Solution:
- Double-check the calculations and steps taken.
- Verify if the solution meets all the given conditions and constraints.
- Consider alternative methods to confirm the solution's validity.

5. Logical Reasoning:
- Use deductive reasoning to arrive at conclusions based on given premises.
- Apply inductive reasoning to identify patterns and make generalizations.
- Employ analogical reasoning by comparing the problem with similar ones solved previously.

6. Making Estimates:
- Use estimation to check the reasonableness of answers.
- Approximations can help simplify complex problems and verify final answers.

7. Using Diagrams and Graphs:
- Visual aids can clarify the problem and reveal insights not immediately obvious from the text.
- Diagrams and graphs can simplify the relationships between variables and aid in understanding geometric and algebraic problems.

8. Formulating Equations: - Translate word problems into algebraic equations. - Identify relevant formulas and apply them correctly to solve for unknown variables.

Exercise

1. What is mathematical reasoning?	2. Why is it important to assess the reasonableness of answers?
3. How can estimation help in assessing the reasonableness of an answer?	4. What is a sanity check in mathematics?
5. How do units of measure play a role in assessing the reasonableness of answers?	6. What is the role of proportional reasoning in assessing answer reasonableness?
7. How can understanding the context of a problem guide the assessment of answer reasonableness?	8. Why is it useful to backtrack through your solution process?
9. How do patterns and relationships contribute to assessing the reasonableness of answers?	10. What is the significance of peer review in assessing answer reasonableness?

Solutions:

1. What is mathematical reasoning? - Mathematical reasoning involves using logic and critical thinking skills to solve problems, make decisions, and justify conclusions. - It includes analyzing problems, developing strategies to solve them, and assessing the validity of solutions.	**2. Why is it important to assess the reasonableness of answers?** - Assessing the reasonableness of answers is important to ensure that solutions to mathematical problems make sense in the given context. - It helps to identify errors in calculations, misunderstandings of the problem, or incorrect assumptions, leading to more accurate and reliable conclusions.
3. How can estimation help in assessing the reasonableness of an answer? - Estimation can provide a rough approximation of what an answer should be, which can then be compared to the actual calculated answer. - If the actual answer is significantly different from the estimated one, it may indicate a mistake in the calculation or reasoning process.	**4. What is a sanity check in mathematics?** - A sanity check in mathematics is a quick, informal test of the validity of an answer or solution. - It involves reviewing the problem and solution to ensure that the answer makes sense and aligns with logical expectations and known facts.
5. How do units of measure play a role in assessing the reasonableness of answers? - Units of measure are crucial for ensuring that answers are reasonable because they provide a scale or context for the solution. - Checking that the units in the answer match the units expected from the problem can help verify that the calculations were performed correctly.	**6. What is the role of proportional reasoning in assessing answer reasonableness?** - Proportional reasoning involves comparing quantities in a way that takes into account the size of differences. - It is useful for assessing the reasonableness of answers by ensuring that relationships and ratios between quantities in the solution are consistent with those in the problem.
7. How can understanding the context of a problem guide the assessment of answer reasonableness? - Understanding the context of a problem helps in setting realistic expectations for what constitutes a reasonable answer. - It allows you to consider real-world constraints and relationships that should be reflected in the solution, helping to judge whether an answer is plausible.	**8. Why is it useful to backtrack through your solution process?** - Backtracking through your solution process is useful for identifying where mistakes may have occurred. - It allows you to review each step for errors or incorrect assumptions and ensures that each part of the process logically leads to the next, confirming the solution's validity.
9. How do patterns and relationships contribute to assessing the reasonableness of answers? - Recognizing patterns and relationships in data or the structure of a problem can provide insights into what types of answers are reasonable.	**10. What is the significance of peer review in assessing answer reasonableness?** - Peer review involves having others check your work, providing an additional layer of scrutiny. - It can reveal misunderstandings, calculation errors, or overlooked details, helping to

- Consistency with known patterns or relationships can serve as a confirmation of an answer's validity, while discrepancies may indicate errors.	ensure that the final answer is as accurate and reasonable as possible.

7-3. Developing Strategies for Problem Solving

1. $5\times[(4+6)-2]+[(2\times2)-1]\times4$ 2. $18\div(9-6)+(5\times2)-(4-2)\times3+2$ 3. $[(10-6)\times(5+1)]-(8\div4)+7$ 4. $8+(8\div4)\times[(9\div3)+5]-4$	52 12 29 20

Exercise

1. What is the first step in developing a strategy for problem solving?	2. How important is it to identify what is known and what needs to be found in a problem?
3. Why should you consider multiple strategies when solving a problem?	4. How can drawing a diagram or model help in problem solving?
5. What role does estimation play in developing problem-solving strategies?	6. How can breaking a problem into smaller, more manageable parts help in solving it?
7. Why is it useful to make an organized list or table when solving problems?	8. How can checking for reasonableness of answers be incorporated into problem-solving strategies?
9. Why is it important to reflect on the problem-solving process after finding a solution?	10. How does persistence play a role in effective problem solving?

Solutions:

1. What is the first step in developing a strategy for problem solving? - The first step in developing a strategy for problem solving is to thoroughly understand the problem. - This means carefully reading the problem statement, identifying what is being asked, and noting any given information and conditions.	**2. How important is it to identify what is known and what needs to be found in a problem?** - It is very important to identify what is known and what needs to be found in a problem. - This helps in organizing the information, determining the relationship between different pieces of data, and focusing efforts on finding the solution to what is unknown.
3. Why should you consider multiple strategies when solving a problem? - Considering multiple strategies when solving a problem is beneficial because it increases the likelihood of finding a solution. - Different approaches can provide unique insights or simplify complex problems, offering alternative paths to the answer.	**4. How can drawing a diagram or model help in problem solving?** - Drawing a diagram or model can help in problem solving by visualizing the problem, making abstract concepts more concrete, and illustrating relationships between different elements. - This can simplify the problem and make it easier to understand and solve.
5. What role does estimation play in developing problem-solving strategies? - Estimation plays a key role in developing problem-solving strategies by providing a quick, approximate solution that can guide the problem-solving process. - It helps in assessing the reasonableness of answers and in making decisions about which strategies to pursue.	**6. How can breaking a problem into smaller, more manageable parts help in solving it?** - Breaking a problem into smaller, more manageable parts can help in solving it by reducing complexity, making it easier to focus on one aspect at a time. - This approach can reveal solutions to sub-problems that contribute to solving the overall problem.
7. Why is it useful to make an organized list or table when solving problems? - Making an organized list or table is useful when solving problems because it helps to systematically arrange and analyze information. - This can highlight patterns, relationships, or missing elements, facilitating a more efficient solution process.	**8. How can checking for reasonableness of answers be incorporated into problem-solving strategies?** - Checking for the reasonableness of answers should be an integral part of problem-solving strategies. - After arriving at a solution, evaluate it against the problem's context, using estimation and logical reasoning to ensure that it makes sense and meets the given conditions.
9. Why is it important to reflect on the problem-solving process after finding a solution? - Reflecting on the problem-solving process after finding a solution is important for learning and improvement. It allows you to understand what strategies worked well, what didn't, and why, helping to refine problem-solving skills and approaches for future problems.	**10. How does persistence play a role in effective problem solving?** - Persistence plays a crucial role in effective problem solving because challenges and setbacks are common. Maintaining determination and continuing to explore different strategies, even when solutions are not immediately apparent, can eventually lead to success.

7-4. Utilizing Estimation Strategies for Computation

1. $[(7+3)\times(4-1)]-[(2\times3)+5]+9$ 2. $[18\div(6\div2)]+[(4\times2)+6]-10$ 3. $[(9-4)\times3]+[(8\div2)\times(3+2)]$ 4. $(6+2)\times[(5-2)+3]-[(4\div2)\times7]+1$	28 10 25 35

Exercise

1. Estimate the sum of 432 and 268 by rounding to the nearest hundred.	2. Estimate the difference between 945 and 487 by rounding to the nearest ten.
3. Estimate the product of 56 and 99 by rounding to the nearest ten.	4. For a quick estimate of the sum 479 + 154, would you round to the nearest ten or hundred? Why?
5. Estimate the product of 78 and 45 by rounding to the nearest ten. Then compare your estimate with the actual product. Which is greater?	6. Estimate the quotient of 789 divided by 26 by rounding the divisor to the nearest ten.
7. You multiply 49 by 25 and get 12,025. Use estimation to check if your answer is reasonable.	8. If a car travels about 30 miles on one gallon of gas, estimate how many gallons of gas you need for a 250-mile trip.
9. Estimate the sum of 18,462 and 76,539 by rounding to the nearest thousand.	10. If you have a rectangle with a length of 93 meters and a width of 58 meters, estimate the area by rounding each measurement to the nearest ten.

Solutions:

1. Estimate the sum of 432 and 268 by rounding to the nearest hundred. - Round 432 to 400 and 268 to 300. - Estimated sum = 400 + 300 = 700.	**2. Estimate the difference between 945 and 487 by rounding to the nearest ten.** - Round 945 to 950 and 487 to 490. - Estimated difference = 950 - 490 = 460.
3. Estimate the product of 56 and 99 by rounding to the nearest ten. - Round 56 to 60 and 99 to 100. - Estimated product = 60 * 100 = 6,000.	**4. For a quick estimate of the sum 479 + 154, would you round to the nearest ten or hundred? Why?** - Nearest ten, because rounding to the nearest hundred (500 + 200 = 700) would give a less accurate estimate than rounding to the nearest ten (480 + 150 = 630).
5. Estimate the product of 78 and 45 by rounding to the nearest ten. Then compare your estimate with the actual product. Which is greater? - Round 78 to 80 and 45 to 40. - Estimated product = 80 * 40 = 3,200. - Actual product = 78 * 45 = 3,510. - The actual product is greater.	**6. Estimate the quotient of 789 divided by 26 by rounding the divisor to the nearest ten.** - Round 26 to 30. - Estimate 789 divided by 30 = 789/30 ≈ 26.
7. You multiply 49 by 25 and get 12,025. Use estimation to check if your answer is reasonable. - Round 49 to 50 and 25 to 20 (for easier multiplication). Estimate = 50 * 20 = 1,000. - Since 12,025 is much higher than 1,000, the original answer is not reasonable.	**8. If a car travels about 30 miles on one gallon of gas, estimate how many gallons of gas you need for a 250-mile trip.** - Estimate = 250 miles / 30 miles per gallon ≈ 8 gallons. - You would need approximately 8 gallons of gas for the trip.
9. Estimate the sum of 18,462 and 76,539 by rounding to the nearest thousand. - Round 18,462 to 18,000 and 76,539 to 77,000. - Estimated sum = 18,000 + 77,000 = 95,000.	**10. If you have a rectangle with a length of 93 meters and a width of 58 meters, estimate the area by rounding each measurement to the nearest ten.** - Round the length to 90 meters and the width to 60 meters. - Estimated area = 90 * 60 = 5,400 square meters.

7-5. Exploring Patterns and Relationships in Numbers and Shapes

1. $[(5+5)\times(3+2)]-[4\div2\times7]+9$ 2. $16-(8\div2)\times(3-1)+(4\times5)$ 3. $(6\times3)-[(4+2)\div2]\times5+6$ 4. $8+12\div6\times3+(4-1)\times3+7$	45 28 9 30

Exercise

1. What is the next number in the sequence: 2, 4, 6, 8, ...?	2. How many degrees are in each angle of an equilateral triangle?
3. In the number pattern 5, 10, 15, __, 25, what number should fill the blank?	4. Is the sum of two odd numbers even or odd? Give an example.
5. What pattern do you notice when you multiply any number by 9? Use 9 x 3 and 9 x 4 as examples.	6. How many lines of symmetry does a square have?
7. What is the relationship between the numbers in the pair (3, 9) in multiplication and division?	8. Identify a pattern in the multiples of 5.
9. What is the square of 5 and what is the square root of 25?	10. If a pattern of dots starts with 1 dot and continues by adding a row of 3 dots in a triangular shape each time, how many dots are there in the 4th pattern?

Solutions:

1. What is the next number in the sequence: 2, 4, 6, 8, ...? - The pattern is adding 2 each time. - The next number is 10.	**2. How many degrees are in each angle of an equilateral triangle?** - An equilateral triangle has all equal angles. - The sum of angles in any triangle is 180 degrees. - Each angle = 180 / 3 = 60 degrees.
3. In the number pattern 5, 10, 15, __, 25, what number should fill the blank? - The pattern increases by 5 each time. - The missing number is 20.	**4. Is the sum of two odd numbers even or odd? Give an example.** - The sum of two odd numbers is always even. - Example: 3 + 5 = 8.
5. What pattern do you notice when you multiply any number by 9? Use 9 x 3 and 9 x 4 as examples. - When you multiply a number by 9, the sum of the digits in the product equals 9. - For example, 9 x 3 = 27 (2 + 7 = 9), and 9 x 4 = 36 (3 + 6 = 9).	**6. How many lines of symmetry does a square have?** - A square has 4 lines of symmetry.
7. What is the relationship between the numbers in the pair (3, 9) in multiplication and division? - 3 multiplied by 3 equals 9, and 9 divided by 3 equals 3. - The numbers show a multiplication and division relationship where one is the square of the other.	**8. Identify a pattern in the multiples of 5.** - All multiples of 5 end in either 0 or 5. - Examples: 10, 15, 20, 25.
9. What is the square of 5 and what is the square root of 25? - The square of 5 is 25 ($5^2 = 25$). - The square root of 25 is 5 ($\sqrt{25} = 5$). - This shows a direct relationship between squares and square roots.	**10. If a pattern of dots starts with 1 dot and continues by adding a row of 3 dots in a triangular shape each time, how many dots are there in the 4th pattern?** - The patterns of dots form triangular numbers: 1, 1+3=4, 4+6=10, 10+9=19. - There are 19 dots in the 4th pattern.